Final Exam Review - Day 2

Ex: A stock is increasing in value by \$8 per share per year. An investor buys shares at a rate of 20 shares per year. How fast is the value of his stock growing when the stock price is \$40 per share and the investor owns 150 shares?

if $n = \#$ shares, $p = \$$ per share, $V =$ Value of shares

$$V = n \cdot p$$

differentiate! $\frac{dV}{dt} = \frac{dn}{dt} \cdot p + n \cdot \frac{dp}{dt}$

$$\frac{dV}{dt} = (20)(40) + (150)(8)$$

$$= 800 + 1200$$

$$= \boxed{\$2{,}000\ /\text{year}}$$

Ex: An object thrown from a cliff with speed after t seconds is $s(t) = 10t + 5$ ft/sec. If the object lands after 9 seconds, how high is the cliff?

distance traveled $= \int_0^9 (10t+5)\,dt$

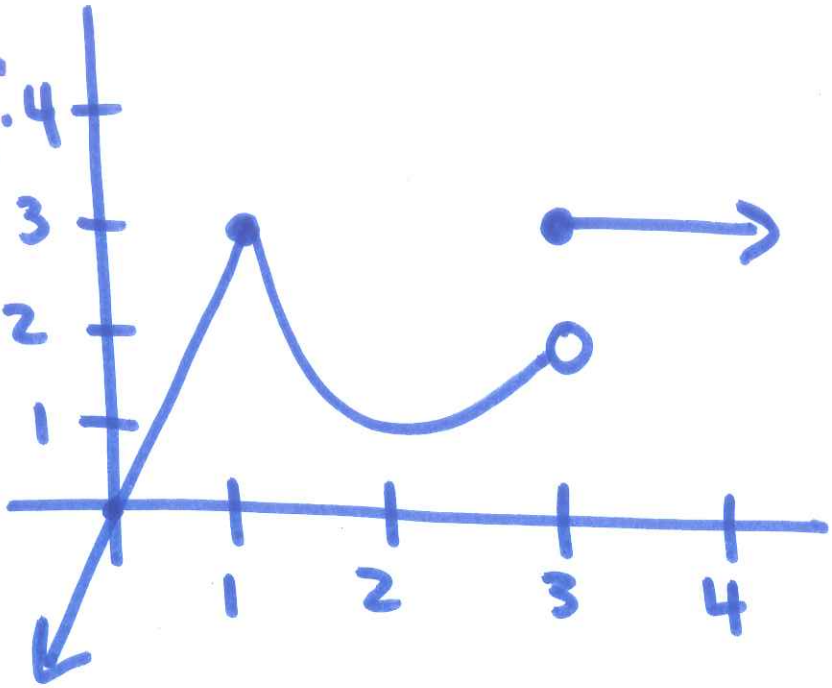
$= (5t^2 + 5t^1)\Big|_0^9$

$= [5(9)^2 + 5(9)] - [5(0)^2 + 5(0)]$

$= 405 + 45 - 0$

$= \boxed{450 \text{ ft}}$

Ex:



graph of $f(x)$.

a) $\lim\limits_{x\to 1} f(x) = \boxed{3}$

b) $\lim\limits_{x\to 3} f(x) = \boxed{DNE}$

c) where is $f(x)$ continuous?

$(-\infty,3) \cup (3,\infty)$

d) where is $f(x)$ differentiable?

$(-\infty,1) \cup (1,3) \cup (3,\infty)$

e) find $f'(\frac{1}{2})$

Slope of tangent line at $f(\frac{1}{2})$

pts $(1,3)$ and $(0,0)$

$f'(\frac{1}{2}) = \frac{3-0}{1-0} = \boxed{3}$

Ex: Compute $2+\underbrace{3+6+9+12+\ldots+210}_{\text{mult. of } 3}$

$$= 2 + \sum_{k=1}^{70} 3k$$

$$= 2 + 3\sum_{k=1}^{70} k$$

$$= 2 + 3\left(\frac{70(71)}{2}\right) = 2+3(2{,}485)$$

$$= \boxed{7{,}457}$$

Ex: $\displaystyle\int_7^{10} x^2\,dx = \sum_{k=1}^{n} \frac{3}{n}\left(A + k\frac{3}{n}\right)^2$

What is A?

$$\int_7^{10} x^2\,dx = \sum_{k=1}^{n} f(x_n)\,\Delta x$$

where $\Delta x = \frac{10-7}{n} = \frac{3}{n}$ and $x_n = 7 + k\frac{3}{n}$

$$\text{thus} = \sum_{k=1}^{n} \left(7 + k\left(\tfrac{3}{n}\right)\right)^2 \left(\tfrac{3}{n}\right)$$

so $\boxed{A = 7}$

Ex: Evaluate $\int_1^{x+3} \sqrt{4t+5}\,dt$

let $u = 4t+5$

$$\frac{du}{dt} = 4 \Rightarrow du = 4dt$$

$$\frac{1}{4}du = dt$$

if $t=1$ then $u=9$

if $t = x+3$ then $u = 4(x+3)+5 = 4x+17$

$$\int_1^{x+3} \sqrt{4t+5}\,dt = \int_9^{4x+17} \sqrt{u} \cdot \frac{1}{4}\,du$$

$$= \int_9^{4x+17} \frac{1}{4} u^{1/2}\,du$$

$$= \left(\frac{1}{6} u^{3/2}\right)\Big|_9^{4x+17}$$

$$= \boxed{\frac{1}{6}(4x+17)^{3/2} - \frac{1}{6}(9)^{3/2}}$$